Non-conservative Effects Produced by Thrust of Jet Engine

By

A. Petre

Polytechnic Institute, Bucuresti, Roumania

It is well known that an aeroelastic phenomenon may occur if an interaction exists between aerodynamic and elastic forces, and in addition massic forces appear for an aeroelastic dynamic phenomenon.

Usually, from the two aerodynamic components, i.e. lift and drag, only the lift effect is taken into account, both components depending on the attack angle. A more accurate physical point of view is to consider two other components of the aerodynamic forces. One, a passive drag, independent in some limits of the attack angle, and another, an active dynamic force, depending on this angle, both attached to the lift surface. Therefore it is possible to refound the classic aeroelasticity by considering the aerodynamic forces as a follower system. Recently the effect of drag on bending-torsion flutter has been shown. It is interesting to point out that the shear force produced by drag could be backward or forward, depending on the position of the propeller engine. If the propelling engine is in the middle of the aeroplane, the shear forces are backward, and if the propelling engines are at the free end of the wing, the shear forces are forward. In the latter case, the engine being attached to the wing, follower forces are also produced.

Let us consider only the case of vertical take-off, case in which the aerodynamic forces of the wing vanish. In this case the wing becomes a simple cantilever beam, assumed to be loaded on the free end by a constant transversal force attached to it and orientated in the direction of the highest rigidity.

For solving the problem, consider the well-known differential equations of bending and torsion for such a beam, assumed for instance to have a constant cross-section

$$EI\,\frac{\partial^4 w}{\partial x^4} + m\,\frac{\partial^2 w}{\partial t^2} + P\,\frac{\partial^2}{\partial x^2}\,[(l-x)\,\varphi] = 0\,,$$

$$GI_d\,\frac{\partial^2 \varphi}{\partial x^2} - J\,\frac{\partial^2 \varphi}{\partial t^2} - P\,(l-x)\,\frac{\partial^2 w}{\partial x^2} = 0\,.$$

Assuming also that at the limit of the stability field the motion is periodic, we can set

$$w = w(x)\cdot e^{i\omega t},$$

$$\varphi = \varphi(x)\cdot e^{i\omega t}$$

and by changing the variable x and the unknown function w into x/l and w/l we obtain another dimensionless differential equations system

$$\frac{d^4 w}{dx^4} - \frac{X}{B_1}\,w + \alpha\,\frac{d^2}{dx^2}\,[(1-x)\,\varphi] = 0\,,$$

$$\frac{d^2 \varphi}{dx^2} + X\varphi - \alpha\,B_2(1-x)\,\frac{d^2 w}{dx^2} = 0\,,$$

in which

$$B_2 = \frac{EI}{GI_d}\,;\quad B_1 = B_2\left(\frac{i}{l}\right)^2;\quad X = \omega^2\,\frac{Jl^2}{GI_d} = \left(\frac{\omega}{\omega_\varrho}\right)^2;\quad \alpha = \frac{Pl^2}{EI}\,,$$

X being the unknown dimensionless frequency and α the unknown critical end force, the beam being specified only by the two dimensionless parameters B_1 and B_2.

It is easy to observe that this equations do not contain end mass effects, and the fact that the end force is a follower force. To clarify the problem it is better to use integral equations. With the aid of Green functions this system becomes

$$\begin{aligned}
w = {} & \frac{X}{B_1}\left[\int_0^x \frac{\xi^2}{6}\,(3x-\xi)\,w\,d\xi + \int_x^1 \frac{x^2}{6}(3\xi - x)\,w\,d\xi\right] \\
& + \alpha\left[\int_0^x (x-\xi)\,(1-x)\,(\varphi-\varphi_n)\,d\xi\right] + \frac{X}{B_1}\,B_4\,\frac{x^2(3-x)}{6}\,w_n,
\end{aligned}$$

$$\begin{aligned}
\varphi = {} & X\left[\int_0^x \xi\varphi\,d\xi + \int_x^1 x\varphi\,d\xi\right] \\
& + \alpha X\,\frac{B_2}{B_1}\left\{\int_0^x \xi\left[(1-\xi)\int_\xi^1 (\bar{\xi}-\xi)\,w\,d\bar{\xi} + B_4(1-\xi)^2\,w_n\right]d\xi\right. \\
& \left. + \int_x^1 x\left[(1-\xi)\int_\xi^1 (\bar{\xi}-\xi)\,w\,d\bar{\xi} + B_4(1-\xi)^2 w_n\right]d\xi\right\} \\
& + \alpha^2 B_2\left[\int_0^x \xi\,(1-\xi)^2\,(\varphi-\varphi_n)\,d\xi + \int_x^1 x\,(1-\xi)^2\,(\varphi-\varphi_n)\,d\xi\right] \\
& + XB_5 x\varphi_n.
\end{aligned}$$

In this equations the fact that the end force is attached to the end cross-section is taken into account by the difference $\varphi - \varphi_n$, and in the last terms the end mass effect appears very clearly, in which

$$B_4 = \frac{M_1}{ml}\,;\ B_5 = \frac{J_1}{Jl} = B_4\left(\frac{i_1}{i}\right)^2,$$

M_1 being the end mass, and J_1 being the moment of inertia of the end mass.

For solving this equations it is necessary to be careful about the modes. It is better to use the extremal property of the eigenvalue in an iterating method. For numerical results, the integral form could be very easily transformed into a linear system, replacing the unknown function w and φ by two vectors with n components. The integral equations become

$$w = \frac{X}{B_1} w_{11} w + \alpha\, \varphi_{11}\varphi\,,$$
$$\varphi = \alpha X w_{21} w + X \varphi_{21}\varphi + \alpha^2 \varphi_{22}\varphi\,,$$

in which w and φ are column matrixes, and w_{11}, φ_{11}, w_{21}, φ_{21} and φ_{22} are square matrixes.

It is interesting to find the lower value of the critical force, no matter whether it corresponds to the first combinations of the modes. The way could be the same as the physical proof of the stability by an initial perturbation. We could take this initial deformation as normed vectors multiplied by an arbitrary amplitude as

$$w = \bar{w} w_n;\ \ \varphi = \bar{\varphi}\varphi_n.$$

From the linear system we could take the last equations

$$w_n = \frac{X}{B_1}\tilde{w}_{11} w_n + \alpha\tilde{\varphi}_{11}\varphi_n\,,$$
$$\varphi_n = \alpha X \tilde{w}_{21} w_n + X\tilde{\varphi}_{21}\varphi_n + \alpha^2\tilde{\varphi}_{22}\varphi_n\,,$$

which yield the secular equation

$$X^2\tilde{w}_{11}\tilde{\varphi}_{21} - X\,[\tilde{\varphi}_{21} + \tilde{w}_{11} + \alpha^2(\tilde{\varphi}_{11}\tilde{w}_{21} - \tilde{w}_{11}\tilde{\varphi}_{22})] + 1 - \alpha^2\tilde{\varphi}_{22} = 0\,.$$

For having stable solutions it is necessary to satisfy the condition

$$[\tilde{\varphi}_{21} + \tilde{w}_{11} + \alpha^2(\tilde{\varphi}_{11}\tilde{w}_{21} - \tilde{w}_{11}\tilde{\varphi}_{22})]^2 - 4\,\tilde{w}_{11}\tilde{\varphi}_{21}(1 - \alpha^2\tilde{\varphi}_{22}) = 0\,,$$

from which we find α, and also the two confounded solutions of X,

$$X = \frac{\tilde{\varphi}_{21} + \tilde{w}_{11} + \alpha^2(\tilde{\varphi}_{11}\tilde{w}_{21} - \tilde{w}_{11}\tilde{\varphi}_{22})}{2\,\tilde{w}_{11}\tilde{\varphi}_{21}}\,.$$

With these results, the linear system formed by the two equations is compatible, and it is possible to find the ratio of the amplitudes

$$\frac{\varphi_n}{w_n} = \frac{1 - \dfrac{X}{B_1}\tilde{w}_{11}}{\alpha\tilde{\varphi}_{11}}\,.$$

Taking $w_n = 1$ and knowing now the whole right side of the complete linear system, we could find two new normed vectors $\bar{\varphi}$ and $\bar{w}$. Repeating this way, we may find with a predicted error the critical value α, the dimensionless frequency X, the modes in flexion and in torsion, and the ratio of the amplitudes of these modes.

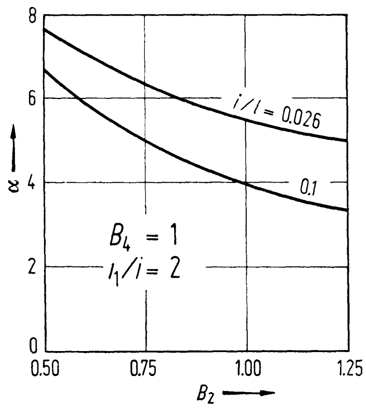

Fig. 1

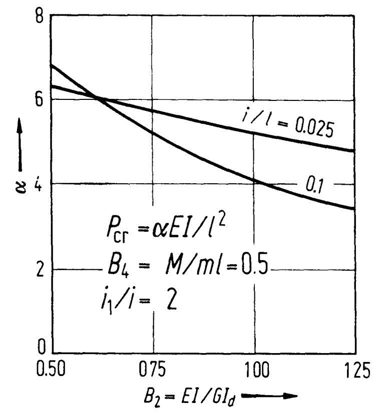

Fig. 2

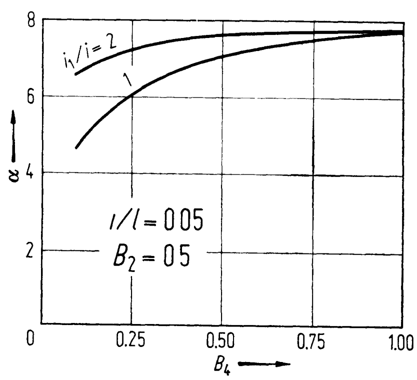

Fig. 3

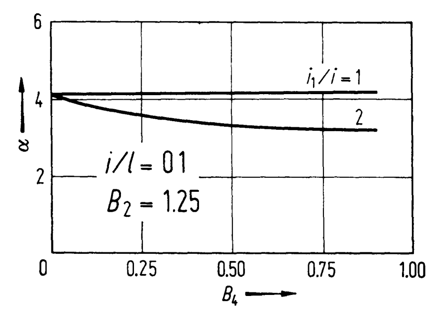

Fig. 4

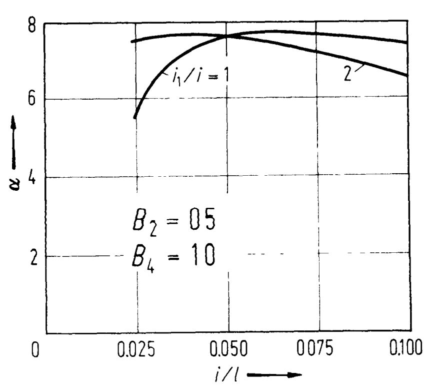

Fig. 5

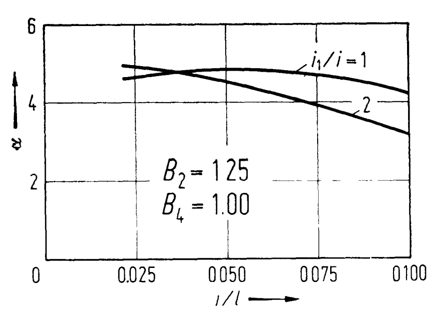

Fig. 6

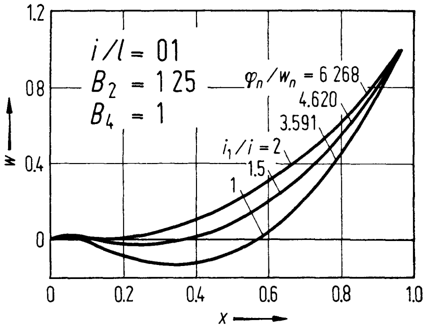

Fig. 7

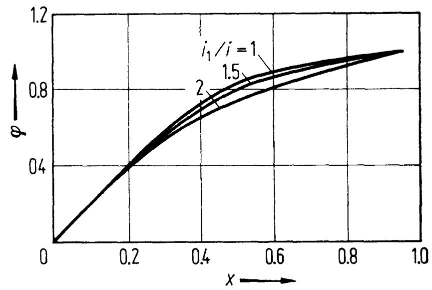

Fig. 8

Figs. 1—6 show some of the results obtained in this way, and Fig. 7 presents some modes in flexion and torsion. It is important to observe that the initial perturbation has no importance for the final results.